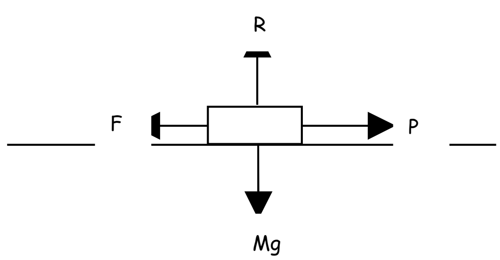
Statics

Friction

Rough and Smooth surfaces

A block of mass M Kg on a horizontal table is acted upon by a force P Newtons. From Newton's Third Law, it is known that equal and opposite forces act on the block and on the plane at right angles to the surfaces in contact.



The force F acts to oppose the motion and this is called the frictional force. If the surface were to be perfectly smooth then the block would accelerate across the surface. In general the surface is unlikely to be smooth and the block would move if the force P was greater than the frictional force.

Limiting Equilibrium

The frictional force in the above situation is not constant, but increases as the force P increases until if reaches a value F_{max}. The block is then on the point of moving and the system is said to be in a state of **limiting equilibrium**.

Coefficient of Friction (μ)

Friction is proportional to the normal reaction and in limiting equilibrium it is given by:

$$F_{max} = \mu R$$

Where μ is the coefficient of friction for the two contact surfaces.

Friction can also be expressed as:

$$F \leq \mu R$$

When friction is less than μR motion will not take place.

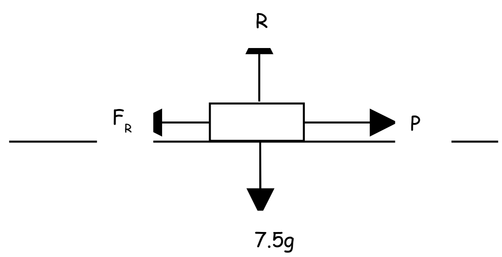
Consider the following points:

1. At the point where friction can't increase any further, motion is about to take place.
2. Note that friction is only dependent upon the nature of the surfaces in contact and not upon the contact area.
3. For perfectly smooth surfaces μ = 0.
4. Friction will never be larger than that necessary to prevent motion.
5. It can be assumed that friction will have a maximum value μR when motion occurs.
6. Friction always acts to oppose the motion of an object and great care must be taken with objects on slopes as friction could be acting either up or down the plane (see examples 7 and 8).

Example 5

A block of mass 7.5Kg rests on a rough horizontal plane, the coefficient of friction between the block and the plane is 0.55. Calculate the frictional force acting on the block when a horizontal force P is applied to the block and the magnitude of any acceleration that may occur.

There is no motion perpendicular to the plane.



a) 15N b)65N

a) Resolving perpendicular to the plane gives:

$$R = 7.5g$$

Friction will act to oppose motion and at its maximum value $F_{max} = \mu R$, therefore:

$$\mu R = 0.55 \times 7.5g$$

$$\mu R = 40.4N$$

In this example it is possible for friction to increase until it reaches a value of 40.4N. There is only a pushing force of 15N therefore friction will prevent motion.

b) Seeing as the pushing force is greater than 40.4N the block will accelerate across the surface. Using the value of μR from above (since we have reached the value of $\mathbf{F_{max}}$) and by setting up an equation of motion for the block we get:

$$F = ma$$

$$65 - 40.4 = 7.5 \times a$$

$$a = 3.28ms^{-2}$$

Note the difference between F and $\mathbf{F_{max}}$: The F is the sum total of all the forces acting in the horizontal direction.

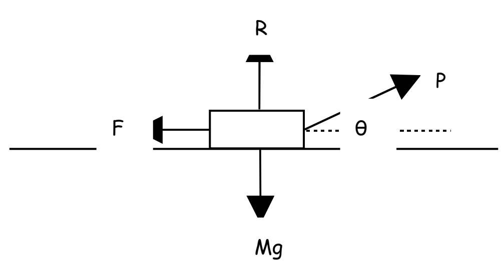
<u>Non Horizontal Forces</u>

When the force P acting on the block of mass M is inclined at an angle to the horizontal, two effects must be considered.

(1) The vertical component of P alters the size of the normal reaction R. One needs to consider the direction of the applied force and its effect on the value of R.

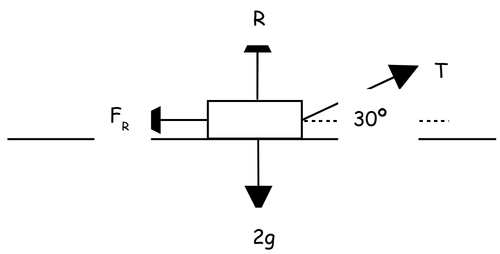
(2) Only the horizontal component of the force P will bring about motion in the block.

It is worth noting at this point that since R has altered there will also be a change in the frictional force (F = μR).



Example 6

A box of mass 2Kg lies on a rough horizontal floor with the coefficient of friction between the floor and the box being 0.5. A light string is attached to the box in order to pull the box across the floor. If the tension in the string is TN, find the value that T must exceed for motion to occur if the string is 30° above the horizontal.



If motion is to take place then $F_R = \mu R$.

Resolving forces perpendicular to the plane gives:

$$2g = R + T \times \sin 30^\circ$$

$$R = 19.6 - 0.5T$$

Using $F_R = \mu R$

$$F_R = 9.8 - 0.25T \quad (1)$$

Resolving parallel to the plane gives:

$$F_R = T \times \cos 30^\circ \quad (2)$$

Equating (1) and (2) gives:

$$9.8 - 0.25T = T\frac{\sqrt{3}}{2}$$

$$T = 8.78\text{N}$$