Statics

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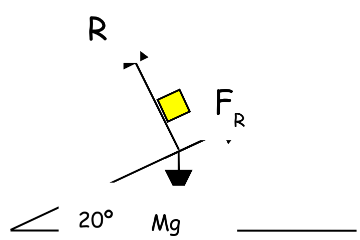
Objects on Inclined Planes

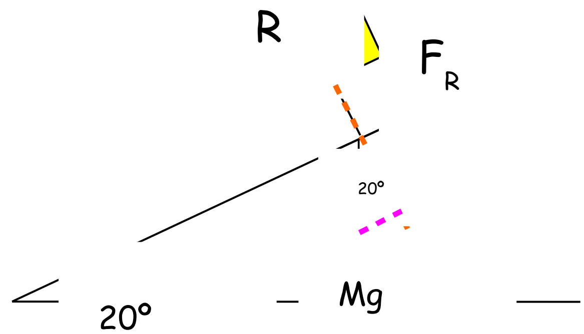
When objects are on inclined planes it is easier to resolve the forces parallel to the plane and perpendicular to the plane. This concept is best shown through an example.

Example 7

A particle of mass MKg rests in equilibrium on a rough plane inclined at an angle 20° to the horizontal. Find the normal reaction R and the frictional force in terms of M and g.



The normal reaction by definition only reacts to the component of the weight force that acts perpendicular to the plane (the orange line in the diagram below). Equally the frictional force is only acting against the component of the weight that is acting parallel to the plane (the pink line).



This leads to the following two statements that will be used over and over again in M1.

$$R = Mg \cos 20°$$

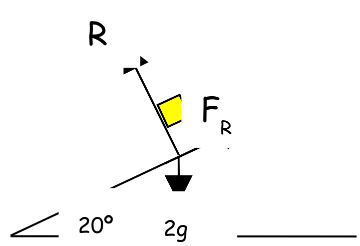
$$F_R = Mg \sin 20°$$

This is only valid if there are no other external forces.

Varying Values of Friction

In the introduction of the coefficient of friction we suggested that friction can vary. Now that we know how to resolve forces parallel and perpendicular to the plane we can use this new skill to explain the point.

Consider the case below with a 2kg mass on a rough surface inclined at an angle of 20°, where the coefficient of friction between the object and the surface is 0.4.



Resolving forces perpendicular to the plane gives:

$$R = 2g \cos 20° = 18.4N$$

Therefore:

$$\mu R = 7.37N$$

Resolving parallel to the plane gives:

$$F_R = 2g \sin 20° = 6.70N$$

The condition $F_R \leq \mu R$ is upheld and as a result there is no motion down the plane.

If the slope was raised to 30°, resolving forces perpendicular to the plane gives:

$$R = 2g \cos 30° = 17.0N$$

Therefore:

$$\mu R = 6.79N$$

Resolving parallel to the plane gives:

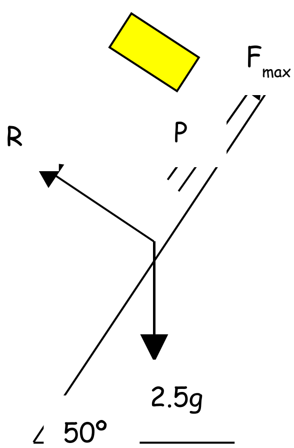
$$F_R = 2g \sin 30° = 9.8N$$

Friction is working against the parallel component of the weight ($mg \sin \theta$). By definition $\sin \theta$ increases as the angle increases therefore friction must increase to prevent motion, but it can only increase to the point where $F = \mu R$. In the second part of the example the condition $F \leq \mu R$ is no longer upheld and therefore the object would slide down the slope. The object in the example above would be in a state of limiting equilibrium for an angle between 20° and 30° (calculate the exact value).

Other external forces on inclined planes

Example 8

A mass of 2.5Kg rests on an inclined plane at 50° to the horizontal, and the coefficient of friction between the mass and the plane is 0.3. Find the force P, acting parallel to the plane, which must be applied to the mass in order to just prevent motion down the plane.
Seeing as the mass is about to slide down the plane, friction must act up the plane.



Resolving the forces parallel to the plane gives:

$$F_{max} + P = 2.5g \sin 50°$$

Resolving perpendicular to the plane:

$$R = 2.5g \cos 50°$$

Using $F_{max} = \mu R$:

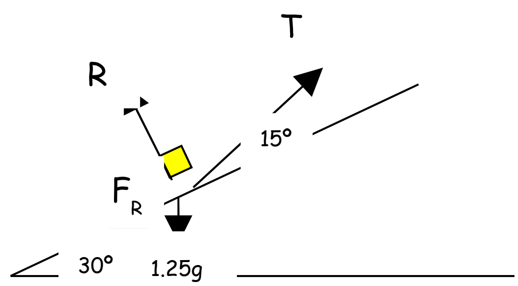
$$0.3 \times (2.5g \cos 50°) + P = 2.5g \sin 50°$$

$$P = 14.0N$$

The next problem introduces the idea of forces acting on a particle on an inclined plane where the force acts at an angle to the plane.

Example 9

A building block of mass 1.25kg is placed on an incline plane at an angle of 30° to the horizontal. The coefficient of friction between the box and the plane is 0.2. The box is kept in equilibrium by a light inextensible string which lies in a vertical plane. The string makes an angle of 15° with the plane. The box is in limiting equilibrium and is about to move up the plane. The tension in the string is T Newtons. Modelling the box as a particle, find the value of T.



Note that the frictional force is acting down the slope as the box is at the point of moving up the plane.
Once again we need to resolve the forces into their components, but this time we must resolve them parallel and perpendicular to the plane.

Resolving parallel to the plane:
(don't forget to resolve the tension force)

$$1.25g \times \sin 30 + F_R = T \times \cos 15$$
$$6.125 + F_R = 0.966T$$
$$F_R = 0.966T - 6.125 \qquad \textbf{(1)}$$

Resolving perpendicular to the plane:

$$1.25g \times \cos 30 = R + T \times \sin 15$$

$$10.61 = R + 0.259T$$

$$R = 10.61 - 0.259T$$

Using $F = \mu R$

$$0.966T - 6.125 = 0.2 \times (10.61 - 0.259T)$$

$$1.0178T = 8.247$$

$$T = 8.10N$$