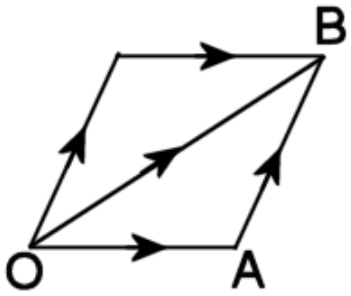
Statics

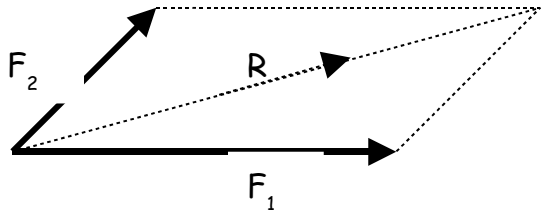
Statics is the study of stationary objects. We will consider a variety of situations where bodies are acted upon by a number of forces. A few of the concepts introduced in our work on vectors will be built upon in this unit.

Resolving Forces

In the vectors unit we were made aware of the fact that the resultant of two vectors is the diagonal of a parallelogram, as highlighted in the diagram below.

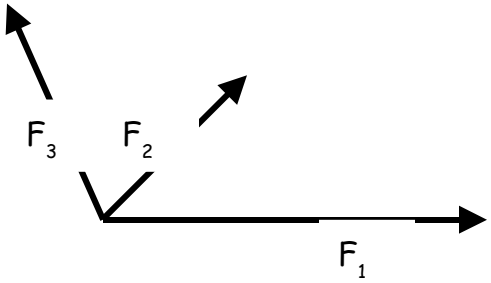


This idea can be applied to forces:



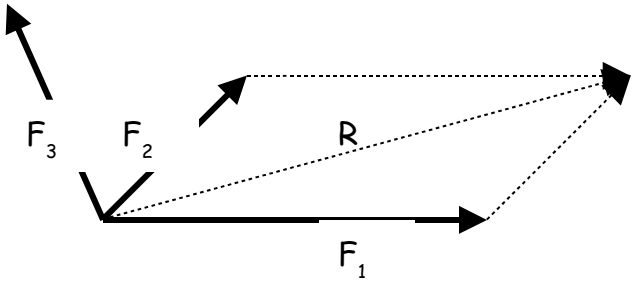
In a real world sense the path R is the direction that a particle would take if it were to be acted upon by the forces F_1 and F_2. This principle can be applied to more than two forces.

Suppose that a particle is acted upon by the forces F_1, F_2 and F_3.

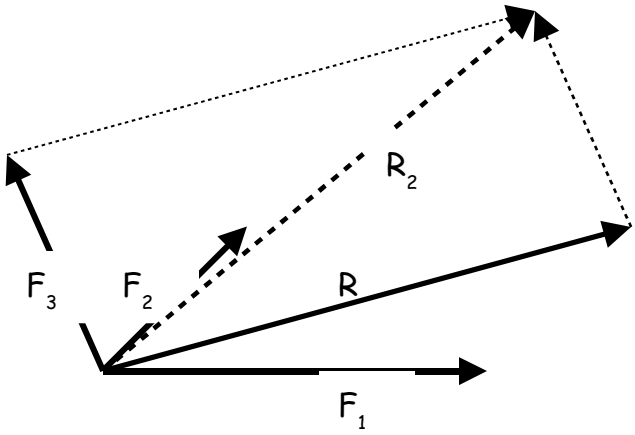


The diagrams below should explain the path that the particle will follow.

Firstly find the resultant of the forces F_1, and F_2 to give R.



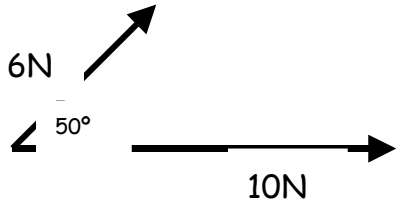
Second find the resultant of R and F_3 .



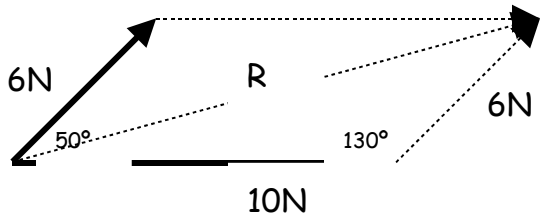
The path R_2 shows the direction of motion of the particle as it is the resultant of the three forces F_1, F_2 and F_3.

Example 1

Two forces act on a particle as outlined in the diagram below. Find the resultant force acting on the object and the angle it makes with the 10N force.



We need to find the magnitude and direction of the force R.



Using Cosine Rule:

$$R^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \cos 130$$

$$R = \sqrt{213.1}$$

Using Sine rule to find the angle that the resultant R makes with the 10N force:

$$\frac{\text{Sin } 130}{R} = \frac{\text{Sin } \theta}{6}$$

$$\text{Sin } \theta = \frac{6 \text{Sin } 130}{\sqrt{213.1}}$$

$$\theta = 18.4^{\circ}$$

Later work will involve more than two forces but the method used is the one we introduced in the vectors unit (resolving into **i** and **j** components).

Questions involving forces will also be given in **i**, **j** notation.

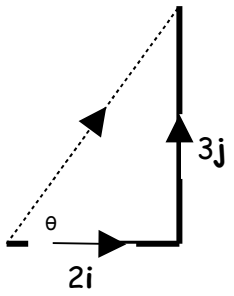
Example 2

Forces E, F and G are applied to a particle. Find the resultant of the three forces in terms of **i** and **j** components. Find the magnitude and the direction of the resulting force.

$E = (7\mathbf{i} - 3\mathbf{j})N, \quad F = (-3\mathbf{i} + 8\mathbf{j})N \quad G = (-2\mathbf{i} - 2\mathbf{j})N$

Since the forces are in Cartesian components, R is found by adding the forces.

Therefore $R = (2\mathbf{i} + 3\mathbf{j})N$



Using Pythagoras to find the magnitude:

$$|R| = \sqrt{2^2 + 3^2}$$

$$|R| = \sqrt{13}$$

And finally the angle θ:

$$\text{Tan}\,\theta = \frac{3}{2}$$

$$\theta = 56.3^{\circ}$$