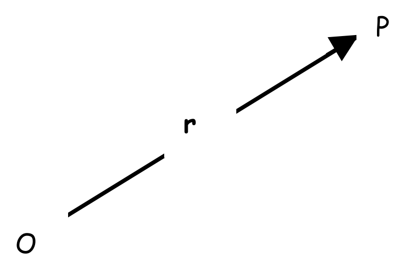
Vectors

Vectors and Mechanics

So far very little in the way of mechanics has been discussed. The positions of particles and their motion can be described by the use of vectors.

Position Vectors

If a particle is moving in a plane, where O is a fixed point, then the position of P is defined by $OP = r$

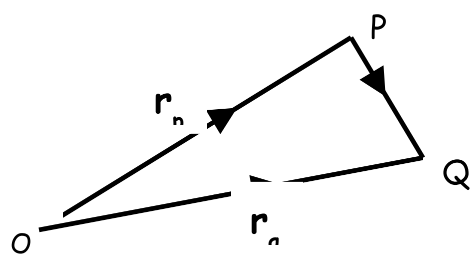


The vector r is known as the **position vector** of P relative to O.

Relative position vectors

As the name suggest we aim to find the vector of one particle relative to another.

Imagine two particles P and Q with position vectors *OP* and *OQ* respectively.



The vector *PQ* gives the position vector of **Q relative to P**. It's called the **relative position vector** (ie how do you get from P to Q).

Velocity as a Vector

We discussed earlier that velocity is a vector quantity but defined more formally:

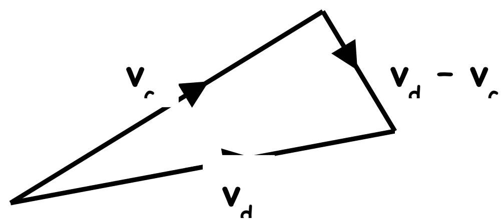
The velocity of a particle is a vector in the direction of motion whose magnitude is equal to the speed of the particle (usually denoted by v)

Relative Velocity

We have discussed relative position vectors, and seeing as velocity is a vector we should be able to consider relative velocity vectors. But what do they mean?

Imagine that two trains are traveling at equal velocities in the same direction at $90kmh^{-1}$. As far as passengers on the trains are concerned they will appear to be stationary. In this case we can say that their relative velocity is zero. If on the other hand the two trains were traveling in the same direction but at velocities of 90kmh^{-1} and 50kmh^{-1} then the relative velocity would be 40kmh^{-1}. This principle can be applied to vectors:

Consider two particles C and D. If their velocities are v_c and v_d respectively then the velocity of D relative to C is $v_d - v_c$.



In real terms the vector $v_d - v_c$ is the velocity vector required to get from C to D. Unfortunately as time passes the particles C and D will get further apart and the vector between them will be a multiple of $v_d - v_c$.

The only other property of the vector $v_d - v_c$ is that if we imagine that C and D are aeroplanes that set off at the same time from the same point, then it is the direction that a passenger on a plane C would look to see plane D.

<u>Acceleration as a Vector</u>

Considering that velocity can be a vector, and that acceleration is the rate of change velocity. It follows that acceleration can be a vector. Using the constant acceleration equations find final velocities.

Example 6

A particle has is moving with velocity (8**i** – 12**j**)ms^{-1} and it experiences an acceleration of (3**i** + 6**j**)ms^{-2} for 4 seconds. Find the final velocity.

The constant acceleration equation needed is **v = u + at**, however I find that it is best to write the equation in words:

Final velocity = initial velocity + (acceleration × time)

The following example uses the equation of motion. A number of vector exam questions will involve ideas from other areas of the M1 course and therefore you need to have a decent grasp of the whole course.

Example 7

A particle Q, of mass 7.5Kg is moving under the action of a constant force F. Initially the velocity of Q is (12**i** – 16**j**)ms^{-1} and 8 seconds later it is (32**i** + 16**j**)ms^{-1}.

a) find, in vector form, the acceleration of Q.

b) calculate the magnitude of F.

a) Given that acceleration is change in velocity over time:

ΔVelocity = (32**i** + 16**j**) - (12**i** – 16**j**)

= (20**i** + 32**j**)

Therefore:

Acceleration = (20**i** + 32**j**)/8 = (2.5**i** + 4**j**)ms^{-2}

b) Using the equation of motion:

$$F = ma$$

$$F = 7.5 \times (2.5\mathbf{i} + 4\mathbf{j})$$

$$F = (18.75\mathbf{i} + 30\mathbf{j})\text{N}$$

The question asks for the magnitude of the force:

$$|F| = 35.4\text{N}$$

The next example uses more concepts but shouldn't cause any problems!

<u>Example 8</u>

A particle A, of mass 3.5Kg is acted upon by two constant forces $(6\mathbf{i} - 3\mathbf{j})$N and $(8\mathbf{i} + 10\mathbf{j})$N.

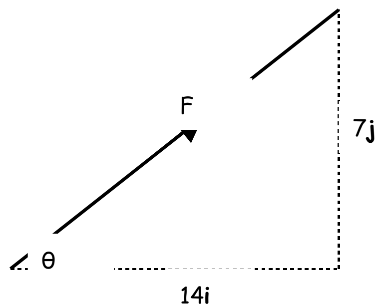
a) Find, in vector form, the resultant force F acting on A.
b) Find, in degrees to 3 sig fig, the angle between F and **i**.
c) Find the magnitude of the acceleration of A.
d) Given that the initial velocity is $(-6\mathbf{i} + 7\mathbf{j})\text{ms}^{-1}$, find the speed of A after 6 seconds.

a) The resultant force is simply the sum of the forces (as they are in **i**,**j** form):

$$F = (6\mathbf{i} - 3\mathbf{j}) + (8\mathbf{i} + 10\mathbf{j})$$

$$F = (14\mathbf{i} + 7\mathbf{j})\text{N}$$

b) The diagram below shows the force F and angle θ:



$\tan\theta = \frac{7}{14}$

$\theta = 26.6^\circ$

c) The acceleration can be found by using the equation of motion for the particle.

$F = ma$

$(14\mathbf{i} + 7\mathbf{j}) = 3.5 \times a$

$a = (4\mathbf{i} + 2\mathbf{j})\text{ms}^{-2}$

d) Seeing as the acceleration is constant we can use constant acceleration equations:

$u = (-6\mathbf{i} + 7\mathbf{j})$, $a = (4\mathbf{i} + 2\mathbf{j})$, $t = 6$

$v = u + at$

$v = (-6\mathbf{i} + 7\mathbf{j}) + 6 \times (4\mathbf{i} + 2\mathbf{j})$

$v = (18\mathbf{i} + 19\mathbf{j})\text{ms}^{-1}$

Speed is the magnitude of the velocity:

$$|v| = \sqrt{(18^2 + 19^2)}$$

$$\text{Speed} = 26.2\text{ms}^{-1}$$