Dynamics

The Principle of the Conservation of Momentum

When a collision occurs between two bodies, A and B, then the force exerted on A by B will be equal and opposite to the force exerted on B by A (by application of Newton's third law). If no other forces are present then the change in momentum in one particle will equate to the loss of momentum in the other particle. Momentum is conserved and therefore the sum of the momentum of the particles before collision must equal the sum of momentum after the collision. This is referred to as the **Principle of Conservation of Momentum**.

If two particles of masses, m_1 and m_2, with initial velocities u_1 and u_2, collide then, given that their final velocities are v_1 and v_2 we can say that:

$$\mathbf{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

The following examples illustrate the principle and once again it is always best to draw a diagram as this will help to avoid mistakes with signs and direction.

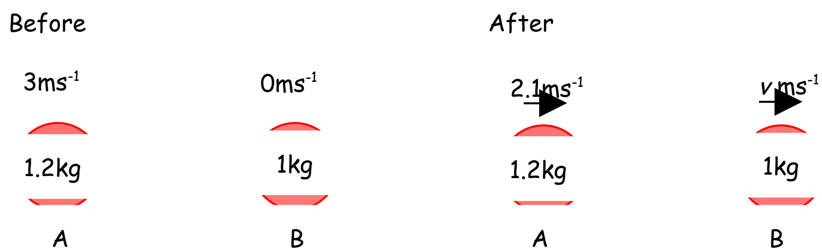
Example 16

Two particles A and B have masses of 1.2kg and 1kg respectively. Particle A is moving towards a stationary particle B with a velocity of 3 ms^{-1}. Immediately after the collision the speed of A is 2.1ms^{-1} and its direction is unchanged. Find:

a) the speed of B after the collision;

b) the magnitude of the impulse exerted on A in the collision.

a)



By conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$3 \times 1.2 = 2.1 \times 1.2 + v$$

$$3.6 - 2.52 = v$$

$$v = 1.08 \text{ ms}^{-1}.$$

b) Impulse = Change in Momentum

$$= m(v - u)$$

$$= 1.2(2.1 - 3)$$

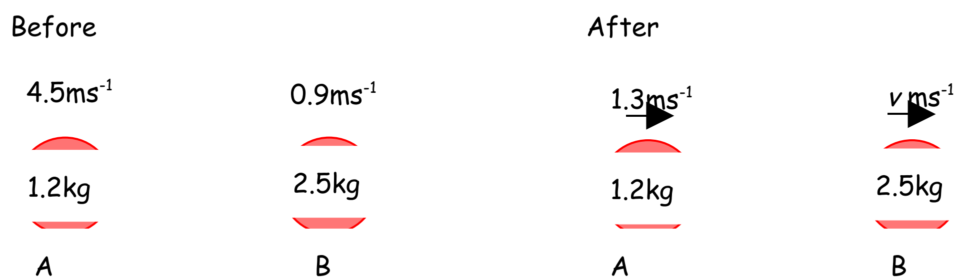
= -1.08 Ns

Example 17

Two small balls A and B have masses 1.2kg and 2.5kg respectively. They are moving in opposite directions on a smooth horizontal surface when they collide directly. Immediately before the collision, the speed of A is $4.5ms^{-1}$ and the speed of B is $0.9ms^{-1}$. The speed of A immediately after the collision is $1.3ms^{-1}$. The direction of A remains unchanged after the collision. Find:

a) the speed of B immediately after the collision;

b) the magnitude of the impulse exerted on B in the collision.



a) By conservation of momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$1.2 \times 4.5 - 2.5 \times 0.9 = 1.2 \times 1.3 + 2.5v$$

$$v = 0.636ms^{-1}$$

b) Impulse = Change in Momentum

$$= m(v - u)$$

$$= 2.5 (0.636 - - 0.9)$$

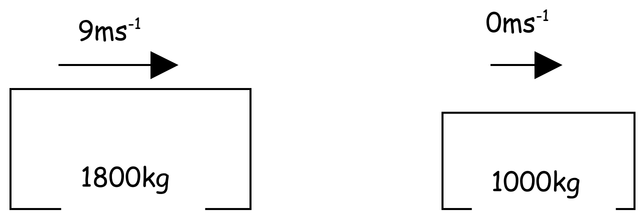
= 3.84Ns

Example 18

A locomotive A, of mass 1800kg is moving along a straight horizontal track with a speed of $9ms^{-1}$. It collides directly with a stationary coal truck, B, of mass 1000kg. In the collision, A and B are coupled and move off together.

a) Find the speed of the combined train.

b) After collision a constant breaking force of magnitude R Newtons is applied. The train comes to rest after 15 seconds. Find the value of R.



a) By conservation of momentum:

$$\mathbf{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

$$9 \times 1800 = 2800 \times v$$

$$v = 5.79ms^{-1}$$

b) The combined train comes to rest in 15 seconds therefore we need to calculate the acceleration for use in an equation of motion.

v = 0, u = 5.79, a = ?, t = 15

Using: $v = u + at$

$$0 = 5.79 + 15a$$
$$a = -0.386ms^{-2}$$

Assuming that the train acts as one body:

Equation of motion:

$$F = ma$$

$$R = -2800 \times 0.386$$

$$R = 1080.8N = 1.10KN$$