Dynamics

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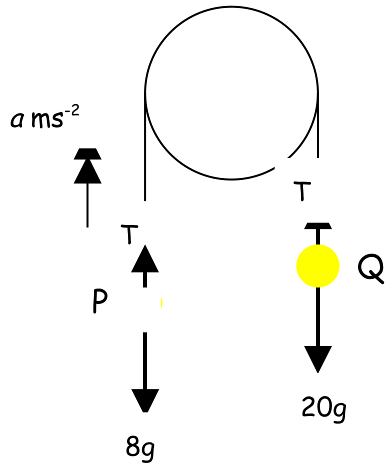
Pulleys

In all questions in M1 the pulley system will be smooth. This implies that the motion of the particles at the end of the string are unaffected by the string passing over the pulley. A further assumption is that the string is light and inextensible. These modelling assumptions make the problem simpler but we can still get pretty realistic answers. If two particles are connected by a string where the string passes over a smooth pulley, then we can assume that the particles will have equal accelerations but in opposite directions.

Example 8

Two particles P and Q are connected by a light inextensible string which passes over a smooth fixed pulley. The system is released from rest. Find:

a) the magnitude of the acceleration;
b) find the tension in the string.



To start the problem set up an equation of motion for particle P. The particle will accelerate upwards hence:

$$F = ma$$

$$T - 8g = 8a \qquad (1)$$

When considering particle Q, its weight will cause it to accelerate down hence the equation of motion is:

$$F = ma$$

$$20g - T = 20a \qquad (2)$$

By adding the two equations the tension will be eliminated:

$$12g = 28a$$

$$a = 4.2ms^{-2}$$

Substituting the value of the acceleration into equation (1) will give the tension.

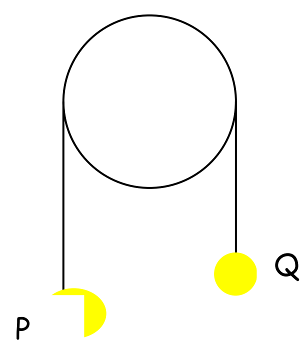
$$T - 8g = 8 \times 4.2$$

$$T = 112N$$

The following example is more algebraic but this does not mean that it is more complicated.

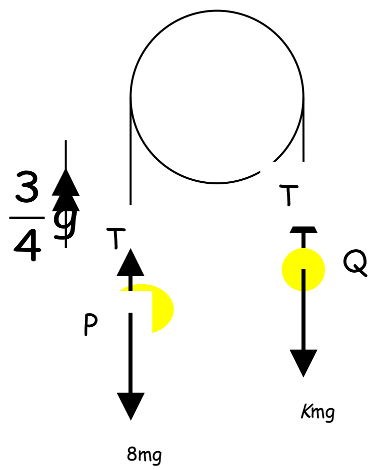
Example 9

Two particles P and Q of masses have masses 8m and *K*m, where *K* > 8. They are connected by a light inextensible string which passes over a smooth fixed pulley. The system is released from rest with the string taut and the hanging parts of the string vertical, as shown below. Initially P has an acceleration of magnitude of $\frac{3}{4}g$.



a) Find, in terms of m and g, the tension, T, in the string.
b) Find the value of *K*.

a) Adding forces to the system:



Setting up an equation of motion for particle P:

$$F = ma$$

$$T - 8mg = 8m \times \frac{3}{4}g$$

$$T = 14mg \quad (1)$$

b) Considering particle Q:

$$Kmg - T = km \times \frac{3}{4}g$$

Using (1):

$$\frac{1}{4}Kmg = 14mg$$

$$K = 56$$

The next problem involves a pulley system where one of the particles is being dragged across a horizontal table as the other particle is falling. The problem will be made more complex when the horizontal table is considered to be rough. The questions are increasing in complexity but the same basic principles apply.

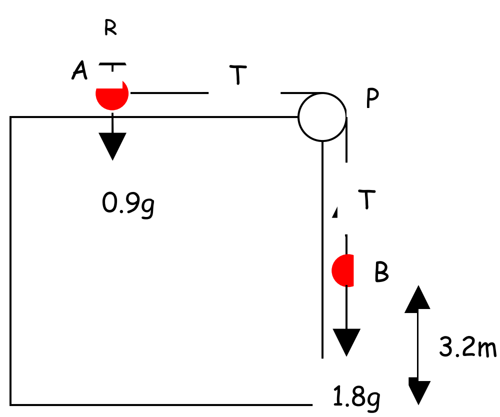
Example 10

A particle A, of mass 0.9kg, rests on smooth horizontal table and is attached to one end of a light inextensible string. The string passes over a smooth pulley P fixed at the edge of the table. The other end of the string is attached to a particle B of mass 1.8kg which hangs freely below the pulley. The system is released from rest with the string taut and B at a height of 3.2m above the ground. In the subsequent motion A does not reach the pulley before B reaches the ground. Find:

a) the tension in the string before B reaches the ground.

b) the time taken by B to reach the ground.

Then, to make the model more realistic, assume that the coefficient of friction between the particle and the table is 0.3. Using this modification find the time taken by B to reach the ground.



a) Setting up equations of motion for the two particles gives:

A F = ma

T = 0.9a

B F = ma

1.8g – T = 1.8a

Adding the two equations gives:

$$1.8g = 2.7a$$

$$a = 6.53ms^{-2}$$

Therefore:

$$T = 0.9 \times 6.53$$

$$T = 5.88N$$

b) The particle B is falling with acceleration $6.53ms^{-2}$. So by using constant acceleration equation we can find the time it takes to reach the floor.

u = 0, a = 6.53, s = 3.2, t = ?

$$s = ut + \frac{1}{2}at^2$$

$$3.2 = \frac{1}{2} \times 6.53 \times t^2$$

$$t = 1.01sec$$

Seeing as the particle is moving friction must be at its maximum value and hence $F_R = \mu R$.

Setting up equations of motion for the two particles with friction included gives:

A $F = ma$ B $F = ma$

$T - F_R = 0.9a$ (1) $1.8g - T = 1.8a$ (2)

Resolving vertically for A:

$$R = 0.9g$$

Using $F_R = \mu R$ $F_R = 0.27g$

Adding equations (1) and (2) gives:

$$1.8g - 0.27g = 2.7a$$

$$a = 5.553\text{ms}^{-2}$$

The new value of acceleration can now be used to calculate the new time:

$u = 0$, $a = 5.553$, $s = 3.2$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

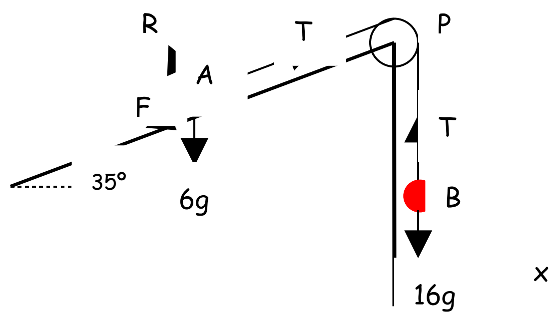
$$3.2 = \frac{1}{2} \times 5.553 \times t^2$$

$$t = 1.07\,sec$$

In the next problem a particle is being pulled up a rough inclined plane by the motion of another particle falling towards a floor. This is very similar to an exam question and would be worth in excess of 10 marks on an M1 paper.

Example 11

A particle, A of mass 6kg, rests on a rough plane inclined at an angle of 35° to the horizontal. The particle is attached to one end of a light inextensible string which lies in a line of greatest slope of the plane and passes over a light smooth pulley P fixed at the top of the plane. The other end of the string is attached to a particle B of mass 16kg. The particles are released from rest with the string taut. The particle B moves down with an acceleration of $\frac{2}{5}g$.



Find:

a) the tension, T, in the string.

b) the coefficient of friction between the plane and A.

a) Setting up equations of motion for A and B gives:

A F = ma

B F = ma

$T - 6g\sin 35° - F_R = 6 \times \frac{2}{5}g$ (1)

$16g - T = 16 \times \frac{2}{5}g$ (2)

Using (2) to find the tension:

$$T = \frac{48}{5}g = 94.08N$$

Resolving vertically for A:

$$R = 6g\cos 35° = 48.166$$

Using $F_R = \mu R$ $F_R = 48.166\mu$

Therefore equation (1) becomes:

$$94.08 - 33.726 - 48.166\mu = \frac{12}{5}g$$

Rearranging for μ:

$$\mu = \frac{60.35 - \frac{12}{5}g}{48.166}$$

$$\mu = 0.765$$

<u>Extension</u>

Assume that in the example above the particle is 5m above a level surface and that after 0.5sec the string breaks. Calculate the total time that the particle B is in flight and the distance that A moves up the plane before it comes to instantaneous rest (assume that it does not reach the pulley).

This is a rather large value for μ and this obviously accounts for the low value for the acceleration.
What is the resultant force on the pulley?